Solution to MHT CET – 2021
23^{rd} September (Shift - 2)

Section I

PHYSICS

1. (C)

The angular velocity

$\omega = 1$ rad/s

$$I = P \cdot \left(\frac{1}{2} I\omega^2\right)$$

$$1 = P \cdot \frac{1}{2} \cdot 1$$

$\therefore\ P = 2$

2. (D)

$r = 0.05\ \text{nm} = 0.05 \times 10^{-9}\ \text{m} = 5 \times 10^{-11}\ \text{m}$

$f = 10^{16}$ r.p.s

$M = IA = ef\pi r^2$

$= 1.6 \times 10^{-19} \times 10^{16} \times 3.14 \times (5 \times 10^{-11})^2$

$= 1.26 \times 10^{-23}\ \text{A–m}^2$

3. (A)

The force due to surface tension at the wall of the capillary is given by

$f_T =$ (surface tension) × (length in contact)

$= T \times 2\pi r_2$

The vertical component of this force is

$f_V = T \times 2\pi r_2 \cos\theta$

where θ is the angle of contact.

Similarly the vertical component of the force at the surface of the rod is

$f'_V = T \times 2\pi r_1 \cos\theta$

Total force $F = f_V + f'_V$

$F = (r_2 + r_1) 2\pi T \cos\theta$

Weight of the liquid in the capillary

$W = \pi\left(r_2^2 - r_1^2\right) h\rho g$

This is balanced by the vertical component of the face due to the surface tension

$\therefore\ \pi\left(r_2^2 - r_1^2\right) h\rho g = \left(r_2 + r_1\right) \times 2\pi T \cos\theta$

Simplifying and solving for h we get,

$$h = \frac{2T\cos\theta}{(r_2 - r_1)\rho g} = \frac{2T}{(r_2 - r_1)\rho g}$$

(For pure water, $\theta = 0°$, $\cos\theta = 1$)

4. (A)

De–Broglie wavelength $\lambda = \frac{1.228}{\sqrt{V}}$ (nm)

If v is increased to 4 V, then λ will become $\frac{1}{\sqrt{4}}$ times or $\frac{1}{2}$ times.

5. (D)

6. (D)

Kinetic energy $= \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$

At $x = \frac{A}{2}$, K.E. $= \frac{1}{2}m\omega^2\left(\frac{3}{4}A^2\right) = \frac{3}{8}m\omega^2 A^2$

$$= \frac{3}{8}m\left(\frac{2\pi}{T}\right)^2 \cdot A^2$$

$$= \frac{3}{8}m \cdot \frac{4\pi^2}{T^2} \cdot A^2$$

$$= \frac{3}{2} \cdot \frac{m\pi^2 A^2}{T^2}$$

7. (D)

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{d^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(ne)^2}{d^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{n^2e^2}{d^2}$$

$$\therefore\ n = \sqrt{\frac{4\pi\varepsilon_0 F d^2}{e^2}}$$

8. (C)

$I_1 = M_1R_1^2,\ \ I_2 = M_2R_2^2$

If m is mass per unit length then

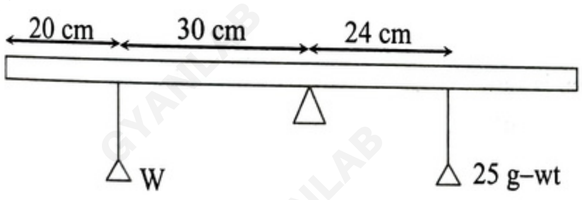
$M_1 = 2\pi R_1 m$ and $M_2 = 2\pi R_2 m$

$$\therefore\ \frac{M_1}{M_2} = \frac{R_1}{R_2}$$

$$\frac{I_1}{I_2} = \frac{M_1}{M_2}\left(\frac{R_1}{R_2}\right)^2 = \frac{R_1}{R_2}\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 = \frac{1}{8}$$

$$\therefore\ \frac{R_1}{R_2} = \frac{1}{2}\ \text{ or }\ \frac{R_2}{R_1} = 2$$

9. (A)



Taking moments about the centre of the scale we get

$W \times 30 = 25 \times 24$

$$\therefore\ W = \frac{25 \times 24}{30} = 20\,\text{g} - \text{wt}$$

10. (D)

When R_1, L_1 and C_1 are connected in series the resonant (angular) frequency is given by

$$\omega_r = \frac{1}{\sqrt{L_1 \cdot C_1}} \qquad ....(1)$$

Also, when R_2, L_2, C_2 are connected in series

then $\omega_r = \dfrac{1}{\sqrt{L_2 \cdot C_2}}$ ....(2)

By (1) and (2) : $L_1C_1 = L_2C_2$

If the two circuits are connected in series then the equivalent inductance is given by $L = L_1 + L_2$ and the equivalent capacitance is given by

$$C = \frac{C_1C_2}{C_1 + C_2}$$

$\therefore$ Resonant (angular) frequency $\omega = \dfrac{1}{\sqrt{LC}}$

$$LC = (L_1 + L_2)\cdot\frac{C_1C_2}{C_1 + C_2}$$

$$= \frac{L_1C_1C_2 + L_2C_1C_2}{C_1 + C_2} \qquad [\because L_1C_1 = L_2C_2]$$

$$= \frac{L_2C_2(C_2 + C_1)}{C_1 + C_2} = L_2C_2$$

$$\therefore \omega = \frac{1}{\sqrt{L_2C_2}} = \omega_r$$

11. (C)

$A = 0.06$ m, K.E. = 10 J, P.E. = 8 J

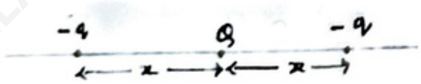
Total energy T.E. = 10 + 8 = 18 J

$$\text{P.E.} = \frac{1}{2}kx^2, \ \text{TE} = \frac{1}{2}kA^2$$

$$\therefore \frac{\text{P.E.}}{\text{T.E.}} = \frac{x^2}{A^2} = \frac{8}{18} = \frac{4}{9}$$

$$\therefore \frac{x}{A} = \frac{2}{3} \text{ or } x = \frac{2}{3}A = \frac{2}{3}\times 0.06 = 0.04 \text{ m}$$

12. (C)



Potential energy of the system is zero.

$$\therefore \frac{1}{4\pi\varepsilon_0}\left(\frac{-q\cdot Q}{x} + \frac{-q\cdot Q +}{x} + \frac{(-q)(-q)}{2x}\right) = 0$$

$$\therefore -2qQ + \frac{q^2}{2} = 0$$

$$\text{or } -2Q + \frac{q^2}{2x} = 0 \quad \therefore 2Q = \frac{q}{2} \text{ or } \frac{Q}{q} = \frac{1}{4}$$

13. (A)

14. (D)

Initial energy $W_1 = \dfrac{1}{2}CV_1^2$

Final energy $W_2 = \frac{1}{2}CV_2^2$

$\therefore \quad \frac{W_2}{W_1} = \left(\frac{V_2}{V_1}\right)^2 = \left(\frac{15}{5}\right)^2 = (3)^2 = 9$

15. (A)
$I = mk^2, \; L = I\omega = mk^2\omega$

$\therefore \quad \omega = \frac{L}{mk^2}$

16. (B)
If I_0 is the intensity of each wave, then the resultant intensity at a point is given by

$I = 4I_0 \cos^2 \frac{\phi}{2}$

At point P, path difference is zero, hence phase difference $\phi = 0$

$\therefore \quad I_p = 4I_0 \cos^2 0 \quad = 4I_0$

At point Q, path difference is $\frac{\lambda}{6}$, hence phase differences is $\frac{2\pi}{6}$ or $\frac{\pi}{3}$.

$\therefore \quad I_Q = 4I_0 \cos^2 \frac{\pi}{6} \quad = 4I_0 \frac{3}{4}$

$\therefore \quad \frac{I_P}{I_Q} = \frac{4}{3}$

17. (C)

18. (A)
Speed of a wave in a string is given by

$V = n\lambda = \sqrt{\frac{T}{m}}$

$\therefore \quad \lambda = \frac{1}{n}\sqrt{\frac{T}{m}}$

$\therefore \quad \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_2}{T_1}}$

Tension at the bottom of the rope = $T_1 = 2$ kg
Tension at the top of the rope $T_2 = 2 + 6 = 8$ kg

$\therefore \quad \lambda_2 = \sqrt{\frac{T_2}{T_1}} \cdot \lambda_1 = \sqrt{\frac{8}{2}} \times 0.06$

$= 2 \times 0.06 = 0.12$ m

19. (C)
For an ideal gas

$R = C_p - C_v = \frac{2}{3}C_v$

$\therefore \quad C_p = C_v + \frac{2}{3}C_v = \frac{5}{3}C_v$

$\therefore \quad \frac{C_p}{C_v} = \frac{5}{3}$

For a monoatomic gas $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$

20. (B)

21. (A)

$u_x = a$ = Horizontal component of the velocity

$u_y = b$ = Vertical component of the velocity

Maximum height $H = \frac{u_y^2}{2g} = \frac{b^2}{2g}$

Range $R = \frac{2u_y u_x}{g} = \frac{2ba}{g}$

$R = 2H \quad \therefore \frac{2ba}{g} = \frac{2b^2}{2g}$

$\therefore \quad b = 2a$

22. (D)

$r = R\left(\frac{\ell_1}{\ell_2} - 1\right) = R'\left(\frac{\ell_1}{\ell_2'} - 1\right);$

$\ell_2 = 1\,m, \ \ell_2' = 1.5\,m \ \ R = 3\Omega, \ \ R' = 6\,\Omega$

$\therefore \ 3\left(\frac{\ell_1}{1} - 1\right) = 6\left(\frac{\ell_1}{1.5} - 1\right)$

Solving we get $\ell_1 = 3$ m

$\therefore \ r = 3\left(\frac{3}{1} - 1\right) = 3 \times 2 = 6\,\Omega$

23. (A)

Since temperature remains constant, there will be no change in r.m.s. velocity.

24. (D)

$F_1 = K_1 x \qquad F_2 = K_2 x \qquad F = (K_1 + K_2)\,x$

$\therefore \ T_1 = 2\pi\sqrt{\frac{m}{K}} \qquad T_2 = 2\pi\sqrt{\frac{m}{K}}$

$\therefore \ T_1^2 = 4\pi^2 \frac{m}{K_1} \qquad T_2^2 = 4\pi^2 \frac{m}{K_2}$

$T^2 = 4\pi^2 \frac{m}{K_1 + K_2}$

$\therefore \ \frac{1}{T^2} = \frac{K_1 + K_2}{4\pi^2 m} = \frac{K_1}{4\pi^2 m} + \frac{K_2}{4\pi^2 m}$

$= \frac{1}{T_1^2} + \frac{1}{T_2^2}$

$= \frac{T_1^2 + T_2^2}{T_1^2 T_2^2}$

$\therefore \ T^2 = \frac{T_1^2 T_2^2}{T_1^2 + T_2^2}$

$\therefore \ T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$

25. (A)

$B_1 = \mu_0 n_1 I_1$ and $B_2 = \mu_0 n_2 I_2$

$n_2 = 5n_1$ $I_2 = 0.2\ I_1$

$\therefore \frac{B_2}{B_1} = \frac{n_2}{n_1} \cdot \frac{I_2}{I_1} = 5 \times 0.2 = 1$

$\therefore B_2 = B_1$

26. (C)

$e = \frac{d\phi}{dt},\ i = \frac{e}{R} = \frac{1}{R} \cdot \frac{d\phi}{dt}$

$\frac{dq}{dt} = \frac{1}{R} \cdot \frac{d\phi}{dt}$

$\therefore dq = \frac{d\phi}{R}$

27. (B)

$P = \frac{1}{3}\rho_1 c_1^2 = \frac{1}{3}\rho_2 c_2^2$

$\therefore \frac{c_1^2}{c_2^2} = \frac{\rho_2}{\rho_1} = 16$

$\therefore \frac{c_1}{c_2} = 4$

28. (B)

$\ell = 31.4\,\text{cm} = 0.314\,\text{m}$

$A = 10^{-3}\ \text{m}^2,\ N = 500$

Self induction L $= \mu_0 n^2 \ell A$

$= \frac{\mu_0 N^2 A}{\ell}$ where $n = \frac{N}{\ell}$

$= \frac{4\pi \times 10^{-7} \times (500)^2 \times 10^{-3}}{0.314}$

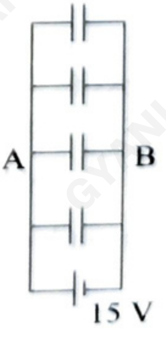
$= \frac{4 \times 3.14 \times 25 \times 10^{-6}}{0.314}$

$= 10^{-3}$ H

29. (A)

The circuit can be drawn as shown

Each capacitor has capacitance of 5 μF.



The capacitors are in parallel. Hence equivalent capacitance

$C = 20\ \mu F$

The charge stored by the combination

$Q = CV = 20 \times 15 = 300\ \mu C$

$\therefore$ The charge on each capacitor is

$\frac{300}{4} = 75\ \mu C$

30. (A)

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For electron and proton change q has same value. If λ' is the wavelength of the proton, then

$$\lambda' = \frac{h}{\sqrt{2Mq \times 9v}}$$

$$\therefore \frac{\lambda'}{\lambda} = \frac{1}{3}\sqrt{\frac{m}{M}}$$

31. (A)

Speed of sound in a medium is independent of pressure.

32. (A)

$\lambda_1 = 5.0$ m and $\lambda_2 = 5.5$ m, v = 300 m/s

$$n_1 = \frac{V}{\lambda_1} = \frac{300}{5} = 60 \text{ Hz}$$

$$n_2 = \frac{V}{\lambda_2} = \frac{300}{5.5} = 54.5 \text{ Hz} \simeq 54 \text{ Hz}$$

Number of beats $= n_1 - n_2 = 60 - 54 = 6$ Hz

33. (B)

Distance of 6th bright band $= \dfrac{6\lambda_1 D}{d}$

Distance of 7th dark band $= \dfrac{6.5\,\lambda_2 D}{d}$

$$\therefore \frac{3\lambda_1 D}{d} = \frac{6.5\lambda_2 D}{d}, \quad \therefore \frac{\lambda_1}{\lambda_2} = \frac{6.5}{6} = \frac{13}{12}$$

34. (B)

$T^2 \alpha\, r^3$

$$\therefore \frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = (4)^3 = 64$$

$$\therefore \frac{T_2}{T_1} = 8 \quad \text{or} \quad T_2 = 8\,T_1 = 8 \times 5 = 40 \text{ hours.}$$

35. (D)

$I_1 = 1.5$ A, $R_1 = 8\Omega$

P.D. across $R_1 = 1.5 \times 8 = 12$ V

$R_2 = 3\Omega$; R_2 is in parallel with R_1 and hence P.D. across it is also 12V.

$$\therefore I_2 = \frac{12}{3} = 4\text{A}$$

Total current $I = I_1 + I_2 = 1.5 + 4 = 5.5$ A

36. (D)

37. (A)

$x = 1\text{ cm}$

$\therefore$ Volume of the cube $v = x^3 = 1\text{ cm}^3$, volume of drop = volume of cube.

$$\frac{4}{3}\pi r^3 = x^3 \quad = 1\text{ cm}^3$$

$$\therefore r^3 = \frac{3}{4\pi} \quad \text{or} \quad r = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}}$$

$$\therefore r^2 = \left(\frac{9}{16\pi^2}\right)^{\frac{1}{3}}$$

$$\text{Surface area of drop} = 4\pi r^2 = 4\pi\left(\frac{9}{16\pi r^2}\right)^{\frac{1}{3}}$$

$$= \left(64\pi^3 \times \frac{9}{16\pi^2}\right)^{\frac{1}{3}}$$

$$= (36\pi)^{\frac{1}{3}}$$

38. (A)

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(\mu_1 - 1)}{R} \quad \text{and}$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\mu_2 - 1}{-R}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R} = \frac{1}{R}[\mu_1 - 1 - \mu_2 + 1] = \frac{\mu_1 - \mu_2}{R}$$

39. (D)

$$Q_1 = \frac{kA(T_1 - T_2)}{L_1} = \frac{k\pi r_1^2 (T_1 - T_2)}{L_1}$$

$$Q_2 = \frac{k\pi r_2^2 (T_1 - T_2)}{L_2}$$

$$\frac{Q_2}{Q_1} = \frac{r_2^2}{r_1^2} \cdot \frac{L_1}{L_2} = (2)^2 \cdot \frac{1}{2} = 2$$

$$\therefore Q_2 = 2Q_1$$

40. (A)

$$E \; \alpha \; \frac{1}{n^2}$$

$$\therefore \frac{E_3}{E_2} = \frac{4}{9} \qquad \therefore E_3 = \frac{4}{9}E_2$$

41. (A)

At resonance, the impedance of the circuit is equal to resistance

$\therefore \; I = \frac{V_R}{R} = \frac{60}{120} = 0.5 \text{ A}$

Inductive reactance $X_L = \frac{V_L}{I} = \frac{40}{0.5} = 80\,\Omega$

$X_L = \omega L$

$\therefore \; L = \frac{X_L}{\omega} = \frac{80}{4\times10^5} = 20\times10^{-5} \text{ H}$

$= 0.2 \text{ mH}$

42. (D)

A body will become weightless at equator, if $R\omega^2 = g$ or $\omega^2 = \frac{g}{R}$

Kinetic energy of the earth $k = \frac{1}{2}I\omega^2$

For a solid sphere $I = \frac{2}{5}MR^2$

(M is mass of the earth)

$\therefore \; k = \frac{1}{2}\times\frac{2}{5}MR^2\times\frac{9}{R} = \frac{1}{5}MgR$

43. (C)

The energy stored in the inductance is given by $U = \frac{1}{2}LI^2$

This energy must be transferred to the capacitor. Energy stored by the capacitor is given by

$w = \frac{1}{2}CV^2$

$\therefore \; \frac{1}{2}CV^2 = \frac{1}{2}LI^2$ $\qquad \therefore \; C = L\left(\frac{I}{V}\right)^2$

44. (B)

Since the some current flows through the second wire, the magnetic field at the same distance will be same. $\therefore B_1 = B_2$

45. (A)

$\Delta I_E = \Delta I_C + \Delta I_B$; $\Delta I_E = 8$ mA, $\Delta I_C = 7.8$ mA

$\therefore \; \Delta I_B = \Delta I_E - \Delta I_C = 8 - 7.8 = 0.2 \text{ mA} = 200\,\mu\text{A}$

46. (D)

At the lower ends of the tube, the total pressure is due to height of water column in the container plus the height of water in the capillary.

$\therefore$ Total pressure is due to 8 + 4 = 12 cm of water.

47. (C)

$\frac{C_p}{C_v} = \frac{3}{2}$

Case I : Isothermal process

$P_1V_1 = P_2V_2$

$PV = P_2 \times 9\,V$

$\therefore\ P_2 = \frac{P}{9}$

Case II : Adiabatic process

$P_2V_2^{\gamma} = P_3V_3^{\gamma}$

$\frac{P}{9}(9V)^{\gamma} = P_3(V)^{\gamma}$

$P_3 = \frac{P}{9}9^{\gamma}\frac{V^{\gamma}}{V^{\gamma}} = \frac{P}{9}9^{3/2} = \frac{P}{9}\times 27 = 3P$

48. (B)

$d = 50\text{cm}$ $\quad \therefore\ r = 25\times10^{-2}\text{ m},\quad f = 2\text{ Hz}$

$a = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2 = 4\pi^2 f^2 r = 4\pi^2\times4\times25\times10^{-2} = 4\pi^2$

49. (A)

$f = 220\text{ Hz},\ v = 330\text{ m/s}$

$v = f\lambda$

$\lambda = \frac{v}{f} = \frac{330}{220} = \frac{3}{2}\text{ m}$

Distance travel by 80 vibrations is

$80\times\frac{3}{2} = 120\text{ m}$

50. (A)

Redistribution of charges takes place.

Charge $q_1 = 3\mu C$ and Charge $q_2 = 8\mu C$

When third charge $q_3 = -5\mu C$ is added to each, then new charges on q_1 and q_2 will be

$q_1' = 3 - 5 = -2\mu C$

and $\quad q_2' = 8 - 5 = 3\mu C$

Now,

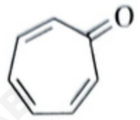
Case I $\quad 40 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{3\times8}{r^2}$

Case II $\quad F = \frac{1}{4\pi\varepsilon_0}\times\frac{(-2\times3)}{r^2}$

$\therefore\quad \frac{F}{40} = \frac{-2\times3}{3\times8} \Rightarrow F = -10\text{ N}$

CHEMISTRY

51. (C)



Tropone (C_7H_6O)
(Non-benzenoid compound)

52. (C)
a = 352 pm, r = ?
For BCC structure,

$$r = \frac{\sqrt{3}}{4}a$$

$$\therefore\ r = \frac{\sqrt{3}}{4} \times 352\ \text{pm}$$

$$= 1.732 \times 88 \qquad = 152.4\ \text{pm}$$

53. (A)
$V_1 = 2 \times 10^{-2}\ m^3$, $V_2 = 3 \times 10^{-2}\ m^3$
$W = -5.09$ kJ $= -5090$ J, $P_{ext} = ?$

$$W = -P_{ext}(V_2 - V_1)$$

$$\therefore\ P_{ext} = -\frac{W}{(V_2 - V_1)}$$

$$= \frac{5090\,J}{(3\times10^{-2})-(2\times10^{-2})\ m^3} \qquad = \frac{5090}{1\times10^{-2}}$$

$$= 5.09 \times 10^5\ Nm^{-2}$$

54. (D)
$C = 0.012$ M, $k = 2.67 \times 10^{-4}$ S cm^{-1}

$$\wedge = \frac{1000\ k}{c}$$

$$= \frac{1000\ cm^3 L^{-1} \times 2.67\times10^{-4}\ S\ cm^{-1}}{0.012\ mol\,L^{-1}}$$

$$\therefore\ \wedge = 22.25\ S\ cm^2\ mol^{-1}$$

55. (C)
22.4 dm^3 of NH_3 $= 6.022 \times 10^{23}$ molecules of NH_3 at STP

$$\therefore\ 67.2\ dm^3\ \text{of}\ NH_3 = \frac{6.022\times10^{23}\times67.2}{22.4}$$

$$= 1.8 \times 10^{22}\ \text{molecules}$$

56. (D)

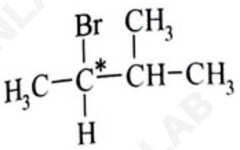
$$:S \equiv C - \ddot{\underset{..}{N}}:$$

$$FC = VE - NE - \left(\frac{BE}{2}\right)$$

$$\therefore\ \text{Formal charge on C atom} = 4 - 0 - \frac{8}{2} \quad = 0$$

57. (B)

58. (B)



2-Bromo-3-methylbutane

59. (C)

$H_2\overset{1}{C}=\overset{2}{C}H-\overset{3}{C}H_2-Cl$

3-Chloropropene

60. (C)

NH_4Cl is a salt of weak base, NH_4OH and strong acid, HCl. Hence, it undergoes hydrolysis. While Na_2SO_4, KCl and KNO_3 are salts of strong acids and strong bases and does not undergo hydrolysis.

61. (B)

62. (C)

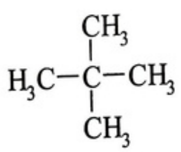
Ni^{2+} (Z = 28) $\Rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^8$

$\therefore$ It has 2 unpaired electrons,

$$\mu = \sqrt{n(n+2)}$$
$$= \sqrt{2(2+2)} = 2.8\text{ BM}$$

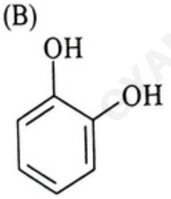
63. (C)

$(CH_3)_4C \rightarrow$



2,2-Dimethylpropane

64. (B)



Common name : Catechol

IUPAC name : Benzene-1,2-diol

65. (B)

$$c = 0.1\text{ M},\ \propto = 1.34\ \% = \frac{1.34}{100} = 1.34 \times 10^{-2}$$

$$K_a = \propto^2 c$$
$$= (1.34 \times 10^{-2})^2 \times 0.1$$
$$= 1.8 \times 10^{-5}$$

66. (A)

$Q = 710\ J,\ \Delta U = 460\ J$

According to first law of thermodynamics,

$\Delta U = Q + W$

$\therefore\ W = \Delta U - Q$

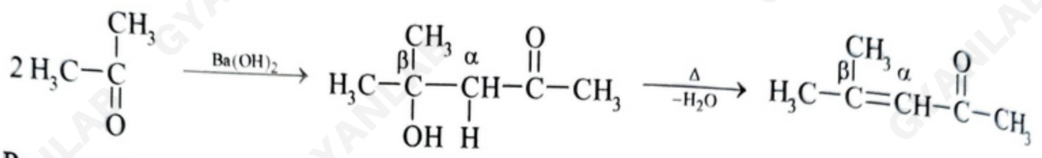
$= 460 - 710 \qquad = -250\ J$

67. (A) **68.** (C) **69.** (B)

70. (B)

During the electrolysis of fused NaCl, a pale green Cl_2 gas is released at anode.

71. (C)



Propanone 4-Hydroxy-4-methyl pentan-2-one 4-Methylpent-3-en-2-one

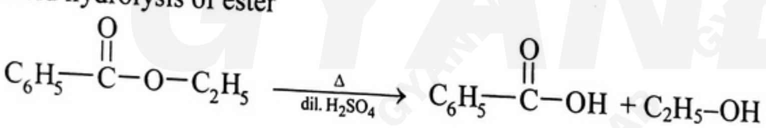
72. (A)

N_2O_5 – Acidic
NO, CO – Neutral
Na_2O – Basic

73. (D) **74.** (B)

75. (C)

Acid hydrolysis of ester



76. (A)

For BCC structure, n = 2

$\rho = 8.55\ g\ cm^{-3},\ M = 93\ g\ mol^{-1},\ a = ?$

$$a^3 = \frac{nM}{\rho N_A}$$

$$= \frac{2\,atoms \times 93\,g\,mol^{-1}}{8.55\,g\,cm^{-3} \times 6.022 \times 10^{-23}\,atoms\,mol^{-1}}$$

$$\therefore\ a^3 = \frac{186 \times 10^{-23}}{51.49}$$

$$= 3.61 \times 10^{-23}\ cm^3$$

$$\therefore\ a = (3.61 \times 10^{23})^{1/3}\ cm$$

77. (B)

$Cu^{2+} \rightarrow 3d^9$ – 1 unpaired electron
$Zn^{2+} \rightarrow 3d^{10}$ – No unpaired electron
$Ti^{3+} \rightarrow 3d^1$ – 1 unpaired electron
$V^{3+} \rightarrow 3d^2$ – 2 unpaired electrons

$\therefore$ Zn^{2+} does not develop spin only magnetic moment due to absence of unpaired electron.

78. (A)

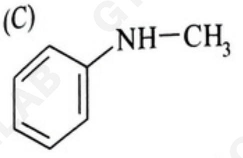
79. (B)

$[Pt(NH_3)_4]^{2+} \Rightarrow Z = 78, X = 2, Y = 8$

EAN of Pt^{2+} $= Z - X + Y$

$= 78 - 2 + 8 \quad = 84$

80. (C)



N-methylbenzenamine

81. (B)

$P_1^0 = 400$ mm Hg, $n_2 = 1$ mol, $n_1 = \frac{36}{18} = 2$ mol

$$\frac{P_1^0 - P_1}{P_1^0} = \frac{n_2}{n_1 + n_2}$$

$$\therefore \frac{400 - P_1}{400} = \frac{1}{3}$$

$\therefore$ $400 - P_1 = 133.33$

$\therefore$ $400 - 133.33 = P_1$

$\therefore$ $P_1 = 266.67$ mm Hg

82. (A)

$pH = 3.6, \ [H^+] = ?$

$pH = -\log_{10}[H^+]$

$\therefore \log_{10}[H^+] = -pH$

$= -3.6$

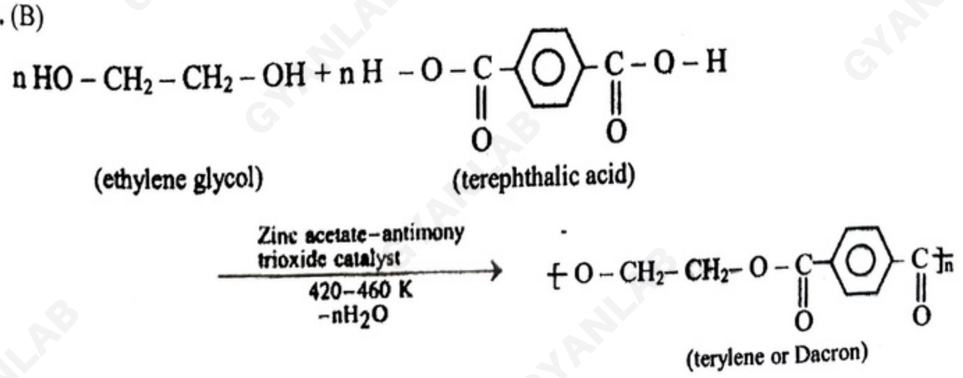
$= -3 - 0.6 - 1 + 1$

$= -4 + 0.4 \quad = \bar{4}.4$

$\therefore \quad [H^+] = \text{Antilog}(\bar{4}.4) \quad = 2.512 \times 10^{-4}$ M

83. (A)

84. (B)



85. (C)

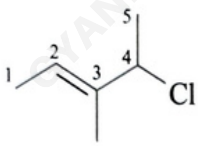
$\lambda = 580 \text{ nm} = 580 \times 10^{-9} \text{ m}, \ \nu = ?$

$$\nu = \frac{c}{\lambda}$$

$$= \frac{3 \times 10^{8} \text{ ms}^{-1}}{580 \times 10^{-9} \text{ m}} = 5.17 \times 10^{14} \text{ Hz}$$

86. (C)

87. (B)



4-Chloro-3-methylpent-2-ene

88. (B)

$\overset{+2}{X}_3(\overset{+5}{Y}\overset{-2}{Z}_4)_2 \rightarrow$ Sum of the oxidation numbers of all atoms is zero.

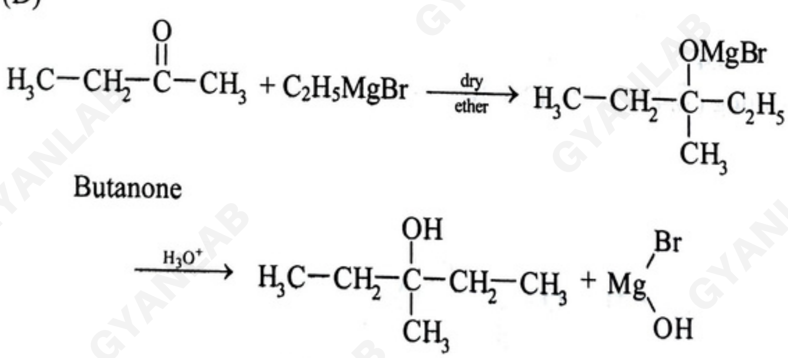
89. (A)

90. (A)

91. (B)

Furan

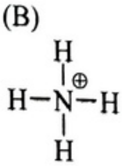
92. (D)



Butanone

3-Methylpentan-3-ol

93. (B)



94. (D)

$$Al^{3+} + 3e^- \longrightarrow Al$$

$I = 1A, t = 9650$ s

$$\text{Mass of product} = \frac{I(A)\times t(s)}{96500(C/mol\ e^-)} \times \text{mole ratio} \times \text{Molar mass of product}$$

$$\therefore \text{Weight of Al deposited} = \frac{1\times 9650}{96500}\times\frac{1}{3}\times 27$$

$$= 0.9\ g$$

95. (D)

$$\text{Rate of reaction} = -\frac{1}{4}\frac{d[NH_3]}{dt} = -\frac{1}{5}\frac{d[O_2]}{dt} = \frac{1}{4}\frac{d[NO]}{dt} = \frac{1}{6}\frac{d[H_2O]}{dt}$$

$\therefore$ Rate of disappearance of NH_3 = Rate of formation of NO

$$= 3.6 \times 10^{-3}\ mol\ L^{-1}\ s^{-1}$$

96. (B)

97. (B)

98. (A)

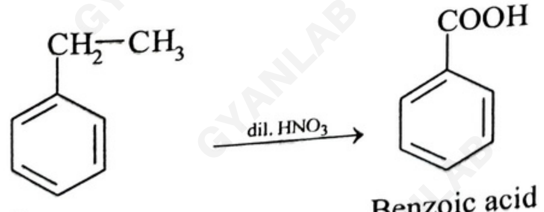
$$\frac{M_{2\,(urea)}}{M_{2\,(A)}} = \frac{W_{2\,(urea)}}{W_{2\,(A)}} \quad [\because \text{solvent is same}]$$

$$\therefore \frac{60}{M_{2\,(A)}} = \frac{6}{9}$$

$$\therefore M_{2\,(A)} = \frac{60\times 9}{6} = 90$$

99. (B)

100. (D)



Ethylbenzene   Benzoic acid

Section II

MATHEMATICS

101.(A)

Let A be the origin.

Then B = (3, 0, 4) and C = (5, −2, 4)

Mid poin of BC = $\left(\frac{3+5}{2}, \frac{0-2}{2}, \frac{4+4}{2}\right)$ i.e. (4, −1, 4)

$\therefore$ Length of median = $\sqrt{(4)^2+(-1)^2+(4)^2} = \sqrt{33}$

102.(C)

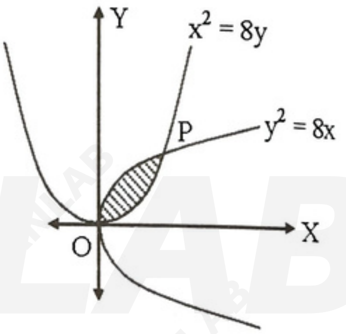
$\cot x = \sqrt{3} \quad \Rightarrow \quad \tan x = \frac{1}{\sqrt{3}}$

$\therefore \quad \frac{1}{\sqrt{3}} = \tan\left(\pi+\frac{\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right) \quad \Rightarrow \frac{1}{\sqrt{3}} = \tan\left(\frac{7\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right)$

103.(B)

Refer figure

Required area is shaded. Point of intersection of given curves are $y^2 = 8x$ and $x^2 = 8y$ i.e.



$\left(\frac{x^2}{8}\right)^2 = 8x \Rightarrow x(x^3 - 512) = 0$

$\therefore$ O ≡ (0, 0) and P ≡ (8, 8)

$\therefore \quad A = \int_0^8 \left(2\sqrt{2}\right)\left(\sqrt{x}\right)dx - \int_0^8 \frac{x^2}{8}dx$

$$= \frac{2\sqrt{2}}{\left(\frac{3}{2}\right)}\left[x^{\frac{3}{2}}\right]_0^8 - \frac{1}{24}\left[x^3\right]_0^8 = \left(\frac{4\sqrt{2}}{3}\right)\left(8\sqrt{8}\right) - \frac{1}{24}(512) = \frac{64}{3} \text{ sq. units}$$

104.(C)

Let p : Two triangles are congruent.

q : Their areas are equal.

Logical form of given statement is $p \rightarrow q$.

Inverse of given statement is $\sim p \rightarrow \sim q$.

Contrapositive of inverse of given statement is $\sim(\sim q) \rightarrow \sim(\sim p)$ i.e. $q \rightarrow p$ i.e.
If areas of two triangles are equal, then they are congruent.

105.(C)

P% amount disappears in one year.

Let initial amount of radium = x_0.

$\therefore$ Amount left after 1 year = $x_0 - \frac{P}{100} \times x_0 = x_0\left(1 - \frac{P}{100}\right)$.

Amount left after 2 years

$$= x_0\left(1-\frac{P}{100}\right)-\frac{P}{100}\times x_0\left(1-\frac{P}{100}\right)$$

$$= x_0\left(1-\frac{P}{100}\right)\left(1-\frac{P}{100}\right) = x_0\left(1-\frac{P}{100}\right)^2$$

106.(D)

Refer figure.

Required area is shaded.

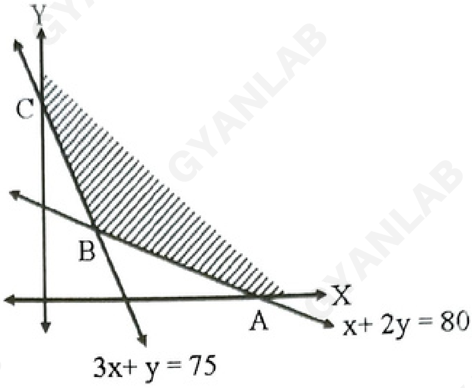
Vertices of feasible region are $A = (80, 0)$; $C = (0, 75)$ and point of intersection of given lines is $B = (14, 33)$

We have to minimize objective function $z = 4x + 6y$

$\therefore\ z_{(A)} = 4(80) + 6(0) = 320$

$z_{(B)} = 4(14) + 6(33) = 254$

$z_{(C)} = 4(0) + 6(75) = 450$



107.(C)

$$\frac{1}{1+\sqrt{2}} = \frac{\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

Hence equations of lines passing through origin and having slopes $\left(\sqrt{2}+1\right)$ and $\left(\sqrt{2}-1\right)$ are

$y = \left(\sqrt{2}+1\right)x$ and $y = \left(\sqrt{2}-1\right)x$.

Their joint equation is $\left[\left(\sqrt{2}+1\right)x-y\right]\left[\left(\sqrt{2}-1\right)x-y\right] = 0$

$\therefore\ x^2 - 2\sqrt{2}\,xy + y^2 = 0$

108.(D)

We have $ax^2 + 2hxy + by^2 = 0$ and let m_1 and m_2 be the slopes of lines.

Now $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

$$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$$

$$= \left(\frac{-2h}{b}\right)^2 - 4\left(\frac{a}{b}\right) = \frac{4h^2}{b^2} - \frac{4a}{b} = \frac{4h^2-4ab}{b^2} = \frac{4h^2-3h^2}{b^2} \quad \text{...[From data given]}$$

$$= \frac{h^2}{b^2}$$

$$\therefore\ m_1 - m_1 = \frac{h}{b}$$

Thus we have $m_1 + m_2 = \frac{-2h}{b}$ and $m_1 - m_2 = \frac{h}{b}$.

Solving, we get $m_1 = \frac{-h}{2b}$ and $m_2 = \frac{-3h}{2b} \Rightarrow m_1 : m_2 = 1 : 3$

109.(B)

$$A = \begin{bmatrix} 5 & 6 & 3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$$

Cofactors of elements in second row are

Cofactor of – 4 $= (-1)^{2+1}\begin{vmatrix} 6 & 3 \\ -7 & 3 \end{vmatrix} = -(18+21) = -39$

Cofactor of 3 $= (-1)^{2+2}\begin{vmatrix} 5 & 3 \\ -4 & 3 \end{vmatrix} = 15 + 12 = 27$

cofactor of 2 $= (-1)^{2+3}\begin{vmatrix} 5 & 6 \\ -4 & -7 \end{vmatrix} = -(-35+24) = 11$

110.(C)

Probability of man speaking truth $= \frac{3}{4}$. ⇒ Probability of telling lies $= \frac{1}{4}$

Probability of a die actually showing 6 $= \dfrac{\left(\frac{1}{6}\times\frac{3}{4}\right)}{\left(\frac{1}{6}\times\frac{3}{4}\right)+\left(\frac{5}{6}\times\frac{1}{4}\right)} = \frac{3}{8}$

111.(C)

$$\bar{a}\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ -1 & 2 & -4 \end{vmatrix} = \hat{i}(-10)-\hat{j}(-9)+\hat{k}(7) = -10\hat{i}+9\hat{j}+7\hat{k}$$

$$\bar{a}\times\bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & 1 & -2 \end{vmatrix} = \hat{i}(-5)-\hat{j}(-3)+\hat{k}(-1) = -5\hat{i}+3\hat{j}-\hat{k}$$

$$(\bar{a}\times\bar{b})\cdot(\bar{a}\times\bar{c}) = \left(-10\hat{i}+9\hat{j}+7\hat{k}\right)\cdot\left(-5\hat{i}+3\hat{j}-\hat{k}\right) = 50+27-7 = 70$$

112.(A)

$y = A\cos\omega t + B\sin\omega t$

$\therefore \frac{dy}{dt} = -A\omega\sin\omega t + B\omega\cos\omega t$

$\frac{d^2y}{dt^2} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t = -\omega^2(A\cos\omega t + B\sin\omega t) = -\omega^2 y$

$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0$

113.(A)

Let a, b, c be the intercepts on the coordinate axes and we have a = b = c.

$\therefore \frac{x}{a}+\frac{y}{a}+\frac{z}{a} = 1$ is required equation of plane.

Since plane passes through point (2, 2, 2), we write

$$\frac{2+2+2}{a} = 1 \quad \Rightarrow a = 6$$

$\therefore$ Equation of plane is $x + y + z = 6$

114.(B)

$$y = x \tan y$$

$$\therefore \frac{dy}{dx} = x \sec^2 y \frac{dy}{dx} + \tan y$$

$$\therefore \left(x \sec^2 y - 1\right)\frac{dy}{dx} = -\tan y$$

$$\therefore \frac{dy}{dx} = \frac{-\tan y}{x \sec^2 y - 1} = \frac{-x \tan y}{x^2 \sec^2 y - x} = \frac{-x \tan y}{x^2\left(1+\tan^2 y\right) - x} = \frac{-x \tan y}{x^2 + x^2 \tan^2 y - x}$$

$$\therefore \frac{dy}{dx} = \frac{-y}{x^2 + y^2 - x} = \frac{y}{x - x^2 - y^2} \quad \ldots[\because y = x \tan y, \text{ given}]$$

115.(B)

Let $$I = \int_0^{\pi} \frac{1}{4+3\cos x}\,dx$$

Put $\tan\frac{x}{2} = t \Rightarrow \cos x = \frac{1-t^2}{1+t^2}$ and $\sec^2\frac{x}{2}\left(\frac{1}{2}\right)dx = dt \Rightarrow dt = \frac{2}{1+t^2}dt$

When $x = 0$, $t = 0$ and when $x = \pi$, $t = \infty$

$$\therefore I = \int_0^{\infty} \frac{1}{4+3\left(\frac{1-t^2}{1+t^2}\right)} \times \frac{2}{1+t^2}\,dt$$

$$= \int_0^{\infty} \frac{\left(1+t^2\right)}{4\left(1+t^2\right)+3\left(1-t^2\right)} \times \frac{2}{1+t^2}\,dt = \int_0^{\infty} \frac{2}{7+t^2}\,dt = \frac{2}{7}\int_0^{\infty} \frac{dt}{1+\left(\frac{t}{\sqrt{7}}\right)^2}$$

$$= \frac{2}{7}\left[\tan^{-1}\left(\frac{t}{7}\right)\right]_0^{\infty} \times \frac{1}{\left(\frac{1}{\sqrt{7}}\right)} = \frac{2}{\sqrt{7}}\left[\tan^{-1}\infty - \tan^{-1}0\right] = \frac{2}{\sqrt{7}} \times \frac{\pi}{2} = \frac{\pi}{\sqrt{7}}$$

116.(C)

$3x + 4y = 9 \quad \Rightarrow 6x + 8y = 18$ and $6x + 8y = 15$ are the two parallel lines.

Distance between the lines $= \dfrac{|18-15|}{\sqrt{(6)^2+(8)^2}} = \dfrac{3}{10} = 0.3$ units

117.(B)

Circle $x^2 + y^2 - 4x + 10y + 20 = 0$ has centre $C_1 = (2, -5)$ and radius $r_1 = \sqrt{4+25-20} = 3$

Circle $x^2 + y^2 + 8x - 6y - 24 = 0$ has centre $C_2 = (-4, 3)$ and radius $r_2 = \sqrt{16+9+24} = 7$

Distance between centres

$= \sqrt{(2+4)^2+(-5-3)^2} = 10$ and $r_1 + r_2 = 3 + 7 = 10$

Thus circles touch each other externally at one point only.

∴ Equation of common tangent is

$(x^2 + y^2 - 4x + 10y + 20) - (x^2 + y^2 + 8x - 6y - 24) = 0$ i.e.

$12x - 16y - 44 = 0 \Rightarrow 3x - 4y - 11 = 0$

118.(B)

We have $\frac{dr}{dt} = 0.01$ and $r = 12$

$A = \pi r^2$

$\therefore \frac{dA}{dt} = \pi(2r)\frac{dr}{dt} = (2\pi)(12)(0.01) = 0.24\,\pi$ sq. cm/sec

119.(D)

Probability of getting $1 = \frac{3}{6} = \frac{1}{2}$

Probability of getting $2 = \frac{2}{6} = \frac{1}{3}$

Probability of getting $5 = \frac{1}{6}$

$\therefore$ Mean $= \Sigma p_i x_i = 2$

x_i	p_i	$p_i x_i$
1	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{3}$	$\frac{2}{3}$
5	$\frac{1}{6}$	$\frac{5}{6}$
Total		2

120.(D)

$y(1+\log x)\frac{dx}{dy} - x\log x = 0$

$\therefore y(1+\log x)\frac{dx}{dy} = x\log x \Rightarrow \frac{dy}{dx} = \frac{y(1+\log x)}{x\log x}$

$\therefore \int\frac{dy}{y} = \int\frac{1+\log x}{x\log x}dx$

$\therefore \log|y| = \log|x\log x| + \log c$

We have $x = e, y = e^2$

$\therefore \log|e^2| = \log|e\log e| + \log c$

$2 = 1 + \log c \Rightarrow \log c = 1 = \log e$

$\therefore \log|y| = \log|x\log x| + \log e$

$y = e\,x\log x$

121.(C)

We know that centroid divides a line joining orthocenter to circumcentre in the ratio 2 : 1.

$\therefore \bar{g} = \frac{\bar{h}+2\bar{p}}{1+2} \Rightarrow 2\bar{p}+\bar{h}-3\bar{g} = 0$

$\therefore x = 2, y = 1, z = -3$ as per data given.

122.(D)

We have $\bar{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$

So coordinates of any point on this line are $[(10\lambda + 11), (-4\lambda - 2), (-11\lambda - 8)]$

Let $P \equiv (2, -1, 5)$ and let M be foot of perpendicular.

$\therefore$ d.r. of PM are $(10\lambda + 9), (-4\lambda - 1), (-11\lambda - 13)$

Since PM is perpendicular to given line, we write

$(10\lambda + 9)(10) + (-4\lambda - 1)(-4) + (-11\lambda - 13)(-11) = 0$

$\therefore \ 100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0 \Rightarrow 237 = -237\lambda \Rightarrow \lambda = -1$

$M \equiv (-10 + 11, 4 - 2, 11 - 8)$ i.e. (1, 2, 3)

123.(D)

$$|A| = \begin{vmatrix} 2 & 3 \\ 10 & 15 \end{vmatrix} = 30 - 30 = 0$$

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 3 \\ 1 & 2 & 3 \end{vmatrix} = 0 \quad \ldots[R_1 \text{ and } R_3 \text{ are identical}]$$

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{vmatrix} = 1(2) - 2(4) + 3(2) = 0$$

$$|D| = \begin{vmatrix} 2 & 4 & 2 \\ 1 & 1 & 0 \\ 1 & 4 & 5 \end{vmatrix} = 2(5) - 4(5) + 2(3) = -4 \neq 0$$

124.(C)

${}^{11}C_4 + {}^{11}C_5 + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r$

We know that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\therefore \ {}^{14}C_r = ({}^{11}C_4 + {}^{11}C_5) + {}^{12}C_6 + {}^{13}C_7$

$= ({}^{12}C_5 + {}^{12}C_6) + {}^{13}C_7$

$= {}^{13}C_6 + {}^{13}C_7 = {}^{14}C_7$

$\therefore \ r = 7$

125.(B)

Let $I = \int \sec^{-1} x \, dx = \int \sec^{-1} x \cdot 1 \, dx$

$= \left(x \sec^{-1} x\right) - \int \frac{x}{x\sqrt{x^2 - 1}} dx$

$= \left(x \sec^{-1} x\right) - \log\left|x + \sqrt{x^2 - 1}\right| + C$

126.(D)

Coefficient of Variation $= \dfrac{\text{Standard Deviation}}{\text{Mean}} \times 100\%$

$= \dfrac{12}{72} \times 100\% = 16.67\ \%$

127.(B)

$x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$

$\therefore \frac{dx}{d\theta} = a(1 + \cos\theta)$ and $\frac{dy}{d\theta} = a(\sin\theta)$

$$\therefore \frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\left(\frac{d\theta}{dx}\right) = \frac{d}{d\theta}\left(\frac{\sin\theta}{1+\cos\theta}\right)\times\frac{1}{\left(\frac{dx}{d\theta}\right)}$$

$$= \frac{(1+\cos\theta)(\cos\theta)-\sin\theta(-\sin\theta)}{(1+\cos\theta)^2\, a(1+\cos\theta)}$$

$$= \frac{\cos\theta+\cos^2\theta+\sin^2\theta}{a(1+\cos\theta)^3} = \frac{1+\cos\theta}{a(1+\cos\theta)^3} = \frac{1}{a(1+\cos\theta)^2}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}} = \frac{1}{a\left(1+\cos\frac{\pi}{2}\right)^2} = \frac{1}{a}$$

128.(C)

We have $2\sin(z + x - y) = \sin(y + z - x) + \sin(x + y - z)$

$\therefore \sin(y + z - x) - \sin(z + x - y) = \sin(z + x - y) - \sin(x + y - z)$

$\therefore 2\cos z \sin(y - x) = 2\cos x \sin(z - y)$

$\therefore \cos z(\sin y\cos x - \cos y\sin x) = \cos x(\sin z\cos y - \cos z\sin y)$

$\therefore \cos x\sin y\cos z - \sin x\cos y\cos z = \cos x\cos y\sin z - \cos x\sin y\cos z.$

Dividing both sides by $\cos x\cos y\cos z$, we get

$\tan y - \tan x = \tan z - \tan y$

$\therefore 2\tan y = \tan x + \tan z$

129.(D)

We have $\sim p \to q$

Inverse of given statement is

$\sim(\sim p) \to \sim q$ i.e. $p \to \sim q$

Negation of inverse of given statement is $\sim(p \to \sim q)$

$\equiv \sim(\sim p \vee \sim q) \equiv p \wedge q$

130.(A)

$$\bar{a}\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$\therefore |\bar{a}\times\bar{b}| = \sqrt{4+4+1} = 3$. Also $|\bar{a}| = \sqrt{4+1+4} = 3$

$\bar{c} - \bar{a} = 2\sqrt{2} \Rightarrow (\bar{c}-\bar{a})^2 = 8$

$\therefore |\bar{c}|^2 + |\bar{a}|^2 - 2\bar{c}\cdot\bar{a} = 8$

$\therefore |\bar{c}|^2 + 9 - 2|\bar{c}| = 8$ ... [$\because \bar{a}\cdot\bar{c} = |\bar{c}|$, given]

$\therefore\ |\bar{c}|^2-2|\bar{c}|+1 \quad = 0 \quad \Rightarrow (|\bar{c}|-1)^2 = 0 \Rightarrow |\bar{c}| = 1$

$|(\bar{a}\times\bar{b})\times\bar{c}| \quad = |\bar{a}\times\bar{b}|\cdot|\bar{c}|\cdot\sin 60^\circ = (3)(1)\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}$

131.(C)

$$\lim_{x\to -\frac{\pi^-}{2}} f(x) = \lim_{x\to -\frac{\pi^-}{2}} -2\sin x = -2\sin\left(-\frac{\pi}{2}\right) = 2$$

$$\lim_{x\to \frac{-\pi^+}{2}} f(x) = \lim_{x\to \frac{-\pi^+}{2}} a\sin x+b = a\sin\left(\frac{-\pi}{2}\right)+b = -a+b$$

Since f(x) is continuous at $x = \frac{-\pi}{2}$, we write

$2 \quad = -a+b \qquad \ldots(1)$

$$\lim_{x\to \frac{\pi^-}{2}} f(x) = \lim_{x\to \frac{\pi^-}{2}} a\sin x+b = a\sin\left(\frac{\pi}{2}\right)+b = a+b$$

$$\lim_{x\to \frac{\pi^+}{2}} f(x) = \lim_{x\to \frac{\pi^+}{2}} \cos x = \cos\left(\frac{\pi}{2}\right) = 0$$

Since f(x) is continuous at $x = \frac{\pi}{2}$, we write

$a+b=0 \qquad \ldots(2)$

From (1) and (2), we get $a=-1, b=1$

$\therefore\ (3a+2b)^3 = (-3+2)^3 \quad = -1$

132.(B)

$\tan^{-1}(2x)+\tan^{-1}(3x) = \frac{\pi}{4}$

$\therefore\ \tan^{-1}\left[\frac{2x+3x}{1-(2x)(3x)}\right] = \frac{\pi}{4} \quad \Rightarrow \tan\frac{\pi}{4} = \frac{5x}{1-6x^2} = 1$

$\therefore\ 6x^2+5x-1 \quad = 0 \quad \Rightarrow (6x-1)(x+1) = 0 \quad \Rightarrow x = -1, \frac{1}{6}$

Since $x > 0$, we get $x = \frac{1}{6}$

133.(C)

Projection $\bar{a}$ on $\bar{b}$ $= \frac{\bar{a}\cdot\bar{b}}{|\bar{b}|} = \frac{(\hat{i}-2\hat{j}+\hat{k})\cdot(2\hat{i}-\hat{j}+\hat{k})}{\sqrt{(2)^2+(-1)^2+(1)^2}} = \frac{2+2+1}{\sqrt{6}} = \frac{5}{\sqrt{6}}$

134.(C)

'X' can take values 0, 1, 2.

Probability of getting no ace card

$$= \frac{^{48}C_2}{^{52}C_2} = \frac{48!}{2!46!} \times \frac{2!50!}{52!} = \frac{188}{221}$$

Probability of getting 1 ace card

$$= \frac{^{4}C_1 \times {}^{48}C_1}{^{52}C_2} = \frac{4 \times 48}{52!} \times 2!50! = \frac{32}{221}$$

Probability of getting 2 ace cards

$$= \frac{^{4}C_2}{^{52}C_2} = \frac{4!}{2!2!} \times \frac{2!50!}{52!} = \frac{1}{221}$$

$$E(X) = \sum p_i x_i$$

$$= (0)\left(\frac{188}{221}\right) + (1)\left(\frac{32}{221}\right) + (2)\left(\frac{1}{221}\right) = \frac{2}{13}$$

135.(C)

$$\frac{d^2y}{dx^2} = \sqrt{\frac{dy}{dx}}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)^2 = \frac{dy}{dx}$$

$\therefore$ order = 2 and degree = 2

136.(A)

Let $I = \int_1^3 \left[\tan^1\left(\frac{x}{x^2-1}\right) + \tan^{-1}\left(\frac{x^2-1}{x}\right)\right] dx$

$$= \int_1^3 \left[\tan^{-1}\left(\frac{x}{x^2-1}\right) + \cot^{-1}\left(\frac{x}{x^2-1}\right)\right] dx$$

$$= \int_1^3 \left(\frac{\pi}{2}\right) dx = \frac{\pi}{2}\int_1^3 dx = \frac{\pi}{2}[x]_1^3 = \pi$$

137.(B)

Amplitude of $(z - 2 - 3i)$ is $\frac{3\pi}{4}$ and we have $z = x + iy$

$\therefore$ Amp $[(x-2) + i(y-3)]$ is $\frac{3\pi}{4}$

$$\therefore \tan^{-1}\left(\frac{y-3}{x-2}\right) = \frac{3\pi}{4} \Rightarrow \tan\left(\frac{3\pi}{4}\right) = \frac{y-3}{x-2}$$

$$\therefore -1 = \frac{y-3}{x-2} \Rightarrow -x + 2 = y - 3 \Rightarrow x + y = 5$$

138.(A)

We have $n = 100$ and probability of getting head $= \frac{1}{2}$

Let $p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

Probability of getting head even number of times =
P[(X = 2) + (X = 4) + … + (X = 100)]

$$= \left[{}^{100}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{98} + \ldots + {}^{100}C_{100}\left(\frac{1}{2}\right)^{100}\left(\frac{1}{2}\right)^{0}\right]$$

$$= \left(\frac{1}{2}\right)^{100}\left[{}^{100}C_2 + {}^{100}C_4 + \ldots + {}^{100}C_{100}\right]$$

$$= \left(\frac{1}{2}\right)^{100}\left[2^{100-1}\right] \quad = \frac{1}{(2)^{100}}\times(2)^{99} \quad = \frac{1}{2}$$

139.(B)

Let $I = \int \frac{\sqrt{x}}{x(x+1)}dx$

Put $x = \tan^2\theta \Rightarrow dx = 2\tan\theta\sec^2\theta\, d\theta$

$$\therefore I = \int \frac{\tan\theta(2\tan\theta\sec^2\theta)}{\tan^2\theta(1+\tan^2\theta)}d\theta$$

$$= 2\int\frac{\sec^2\theta}{\sec^2\theta}d\theta = 2\int d\theta \quad = 2\theta$$

$$= 2\tan^{-1}\sqrt{x} + c$$

Comparing with given data, $k = 2, m = \sqrt{x}$

140.(A)

$$y = \sqrt{2}\sin\left(2x+\frac{\pi}{4}\right)$$

$$\frac{dy}{dx} = \sqrt{2}\cos\left(2x+\frac{\pi}{4}\right)(2)$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = (2\sqrt{2})\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right) = 2\sqrt{2}\left(-\sin\frac{\pi}{4}\right) \quad = -2$$

When $x = \frac{\pi}{4}$, $y = \sqrt{2}\sin\left(\frac{\pi}{2}+\frac{\pi}{4}\right) = 1$

Hence equation of required tangent is

$$(y-1) = -2\left(x-\frac{\pi}{4}\right) \Rightarrow 2x + y - \frac{\pi}{2} - 1 \quad = 0$$

141.(C)

$$\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$$

$\therefore$ sin A (sin B cos C − cos B sin C) = sin C (sin A cos B − cos A sin B)
$\therefore$ sin A sin B cos C + cos A sin B sin C = 2 sin A cos B sin C
$\therefore$ sin B(sin A cos C + cos A sin C) = 2 sin A cos B sin C
$\therefore$ sin B[sin (A + C)] = 2 sin A cos B sin C
$\therefore$ sin B sin B = 2 sin A cos B sin C …[$\because$ A + B + C = π]

$\therefore \dfrac{1}{2\cos B} = \dfrac{\sin A}{\sin B} \times \dfrac{\sin C}{\sin B}$

By sine rule, we know that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\therefore \dfrac{1}{2\left(\dfrac{c^2+a^2-b^2}{2ac}\right)} = \dfrac{a}{b} \times \dfrac{c}{b} \Rightarrow \dfrac{ac}{c^2+a^2-b^2} = \dfrac{ac}{b^2}$

$\therefore c^2 + a^2 - b^2 = b^2 \Rightarrow 2b^2 = a^2 + c^2$

142.(C)

$\cos(x+y)\dfrac{dy}{dx} = 1$

Put $x + y = V \Rightarrow 1 + \dfrac{dy}{dx} = \dfrac{dV}{dx}$

$\therefore \cos V\left(\dfrac{dV}{dx} - 1\right) = 1 \Rightarrow \cos V\left(\dfrac{dV}{dx}\right) = 1 + \cos V$

$\therefore \int \dfrac{\cos V}{1+\cos V} dV = \int dx$

$\therefore \int\left[\dfrac{1+\cos V}{1+\cos V} - \dfrac{1}{1+\cos V}\right]dV = \int dx \Rightarrow \int dV - \dfrac{1}{2}\int \sec^2 \dfrac{V}{2} dV = \int dx$

$\therefore V - \dfrac{1}{2}\dfrac{\tan\left(\dfrac{V}{2}\right)}{\left(\dfrac{1}{2}\right)} = x + c \Rightarrow V - \tan\left(\dfrac{V}{2}\right) = x + c \Rightarrow x + y - \tan\left(\dfrac{x+y}{2}\right) = x + c$

$y = \tan\left(\dfrac{x+y}{2}\right) + c$

143.(B)

$f(x) = \cot^{-1}\left[(\cos 2x)^{\frac{1}{2}}\right] = \cot^{-1}\left(\sqrt{\cos 2x}\right)$

$\therefore f'(x) = \dfrac{-1}{1+\left(\sqrt{\cos 2x}\right)^2} \times \dfrac{d}{dx}\left(\sqrt{\cos 2x}\right)$

$= \dfrac{-1}{1+\cos 2x} \times \dfrac{1}{2\sqrt{\cos 2x}} \times (-2\sin 2x) = \dfrac{2\sin 2x}{2(1+\cos 2x)\sqrt{\cos 2x}}$

$\therefore \left[f'(x)\right]_{x=\frac{\pi}{6}} = \dfrac{\sin\left(\dfrac{\pi}{3}\right)}{\left(1+\cos\dfrac{\pi}{3}\right)\sqrt{\cos\dfrac{\pi}{3}}} = \left(\dfrac{2}{3}\right)^{\frac{1}{2}}$

144.(B)

$f(x) = \log_{10}\left(x^2 - 5x + 6\right)$

$\therefore x^2 - 5x + 6 > 0 \Rightarrow (x-2)(x-3) > 0 \Rightarrow x > 3$ or $x < 2$

$\therefore x \in (-\infty, 2) \cup (3, \infty)$

145.(C)
Since given vectors are coplanar, we write

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

$\therefore\ 2(10 + 3\lambda) + 1(5 + 9) + 1(\lambda - 6) = 0$

$\therefore\ 20 + 6\lambda + 14 + \lambda - 6 = 0 \Rightarrow 7\lambda + 28 = 0 \quad \Rightarrow \lambda = -4$

Put $x = -4$ in all options.

(A) $8 - 12 = -4 \neq 6$

(B) $16 - 8 = 8 \neq 4$

(C) $16 - 12 = 4$

(D) $16 - 8 = 8 \neq 6$

146.(D)

Area $= \frac{ac\sin B}{2} \Rightarrow 10\sqrt{3} = \frac{ac\sin 60}{2}$

$10\sqrt{3} = \frac{ac\sqrt{3}}{4} \quad \Rightarrow \quad ac = 40$

Now

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos 60 \times 2ac = a^2 + c^2 - b^2$

$\therefore\ \frac{1}{2} \times 2 \times 40 = (a + c)^2 - 2ac - b^2$

$\therefore\ 40 = (20 - b)^2 - (2 \times 40) - b^2$ ... [a + b + c = 20, given]

$= 400 + b^2 - 40b - 80 - b^2$

$\therefore\ 40b = 280 \Rightarrow b = 7$

147.(D)

Let $y = m_1x$ and $y = m_2x$ be the lines represented by the equation. $ax^2 + 2hxy + by^2 = 0$

Then, $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

We have, $\frac{m_1}{m_2} = \frac{5}{3} \Rightarrow m_1 \Rightarrow \frac{5m_2}{3}$

$\therefore\ \frac{5m_2}{3} + m_2 = \frac{-2h}{b}$ and $\left(\frac{5m_2}{3}\right)m_2 = \frac{a}{b}$

$\therefore\ \frac{8m_2}{3} = \frac{-2h}{b} \Rightarrow m_2 = \frac{-3h}{4b}$ and $m_2^2 = \frac{3a}{5b}$

$\left(\frac{-3h}{4b}\right)^2 = \frac{3a}{5b} \Rightarrow \frac{9h^2}{16b^2} = \frac{3a}{5b}$

$\therefore\ \frac{h^2}{ab} = \frac{16}{15}$

148.(A)

We have $f(x) = \dfrac{1 - x + x^2}{1 + x + x^2}$

$$\therefore\ f'(x) = \frac{(1+x+x^2)(2x-1)-(1-x+x^2)(2x+1)}{(1+x+x^2)^2}$$

$$= \frac{(2x + 2x^2 + 2x^3 - x - 1 - x^2) - (2x - 2x^2 + 2x^3 + 1 - x + x^2)}{(1+x+x^2)^2}$$

$$= \frac{(x + x^2 + 2x^3 - 1) - (x - x^2 + 2x^3 + 1)}{(1+x+x^2)^2}$$

$= \dfrac{2x^2 - 2}{(1+x+x^2)^2}$ and when $f'(x) = 0$, we get

$2(x^2 - 1) = 0 \quad \Rightarrow \quad x = \pm 1$

When $x = 1$, $f(x) = \dfrac{1}{3}$ and when $x = -1$, $f(x) = 3$

Hence minimum value of f(x) is $\dfrac{1}{3}$.

149.(A)

Let $I = \displaystyle\int \frac{dx}{\cos x\sqrt{\cos 2x}}$

$$= \int \frac{dx}{\cos x \cdot \cos x\sqrt{\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}}} = \int \frac{dx}{\cos^2 x\sqrt{1 - \tan^2 x}}$$

$$I = \int \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}}\,dx$$

Put $\tan x = t \Rightarrow \sec^2 dx = dt$

$$\therefore\ I = \int \frac{dt}{\sqrt{1 - t^2}} = \sin^{-1}(t) + c = \sin^{-1}(\tan x) + c$$

150.(A)

Given f is a the p.d.f. of a r.v.x.

$$\therefore\ \int_0^1 f(x)\,dx = 1 \Rightarrow \int_0^1 K(x - x^2)\,dx = 1$$

$$K\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1 \Rightarrow K\left(\frac{1}{2} - \frac{1}{3}\right) = 1 \Rightarrow \frac{K}{6} = 1$$

$K = 6$

$$\therefore\ P\left(X < \frac{1}{2}\right) = \int_0^{\frac{1}{2}} 6\left(x - x^2\right)dx = \left[\frac{6x^2}{2}\right]_0^1 - \left[\frac{6x^3}{3}\right]_0^1$$

$$= \left[3x^2 - 2x^3\right]_0^{\frac{1}{2}} = \frac{3}{4} - \frac{2}{8} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$